Chapt 6 X-rays

6.1. Discovery of X-rays & their wave nature
6.2. Mechanisms for producing X-rays
6.3. Compton scattering
6.4. Absorption of X-rays
6.5 . Summary

6.1. Discovery of X-rays & their wave nature

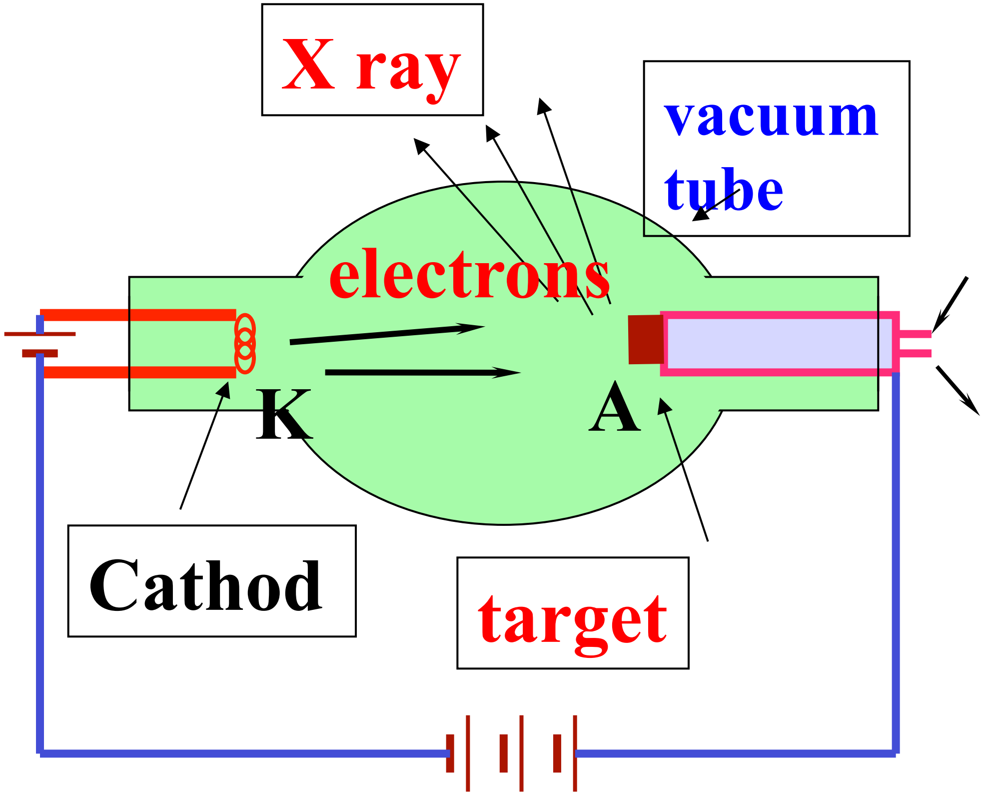
6.1.1.Discovery of X-rays

- **In 1895 , Roentgen performed an experim.on a gas discharge in a chathode ray tube in a darkroom, found a fluorescent screen exhibited slight fluorescence by a mysterious new rays:**
  - **Traveled straight**
  - **Strong penetrating power (neither reflection nor refraction)**
  - **Not deflected by a Magnetic field**
  - ***x -rays***

- **People realized that:**
  - **x –rays are energetic em wave with strong penetrating power**
  - **X-rays are EM wave with short wave length**

    (L) :**0.001nm — 1 nm**
  - Soft x ray: L> **0.1nm**
  - Hard x rays: L< **0.1nm**

6.1.2. X-ray tube (1)



X ray tube

electrons from **heated K are accellerated by E and strike on the Anode , producing x-rays**

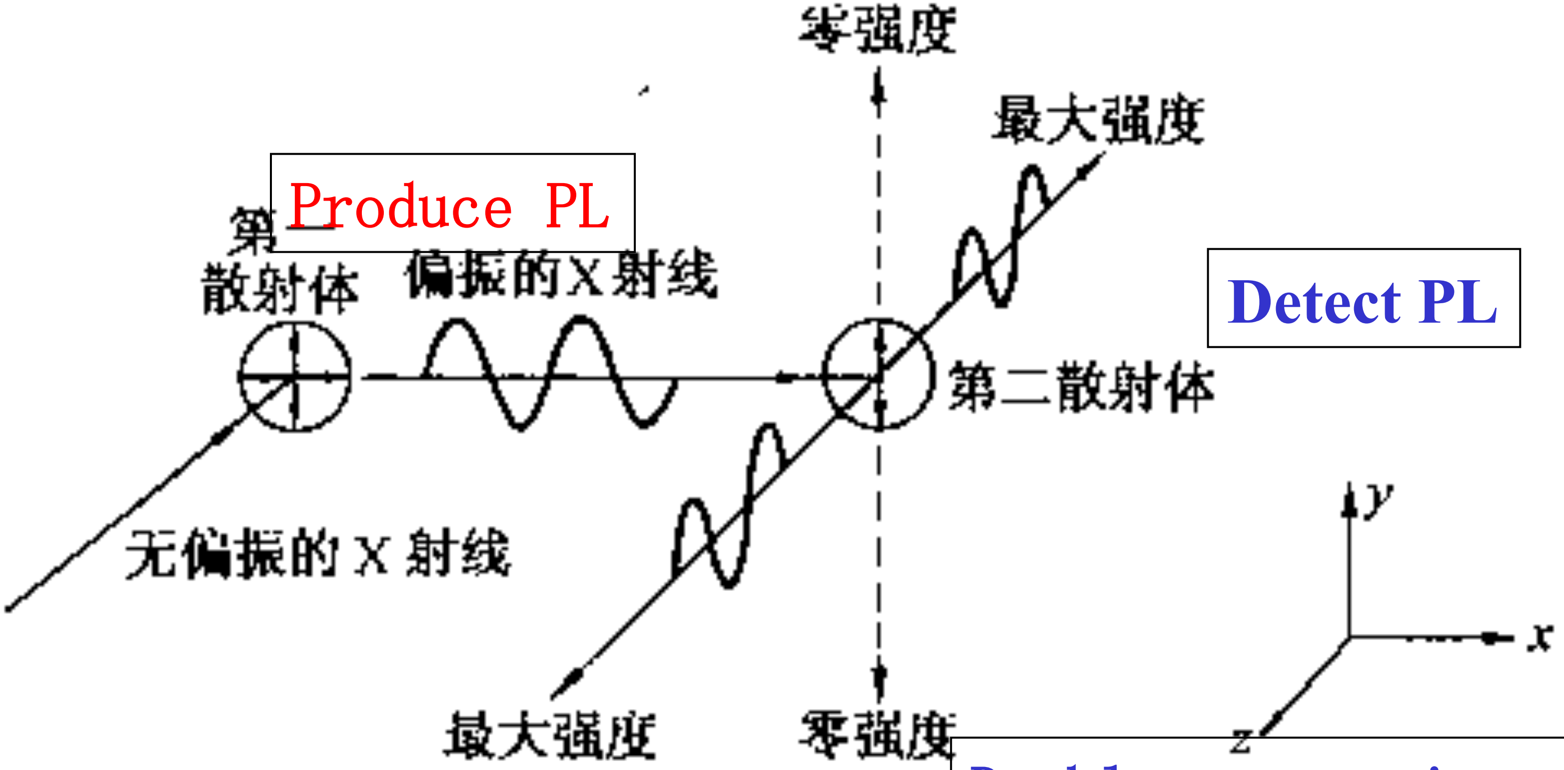
6.1.3. Wave nature of X-rays

- Accelerating charged particles→radiate EM waves
- In x-ray tube, high speed Es stop on the Anode→ radiate EM waves
- **X-rays**←→EM waves
- Charaters of wave
  - polarization, diffraction

6.1.4 Polarization of X-rays

- Transverse wave: oscillating direction is perpendicular to the propagation direction
  - EM wave is transverse wave (E,K)
  - PL concept only holds for transverse wave
- **Linear PL:** ***E oscillates along a fixed direction***
- **Circular PL:** ***E moves along a circle***

Double scattering experiment



Produce PL

Detect PL

Double scattering

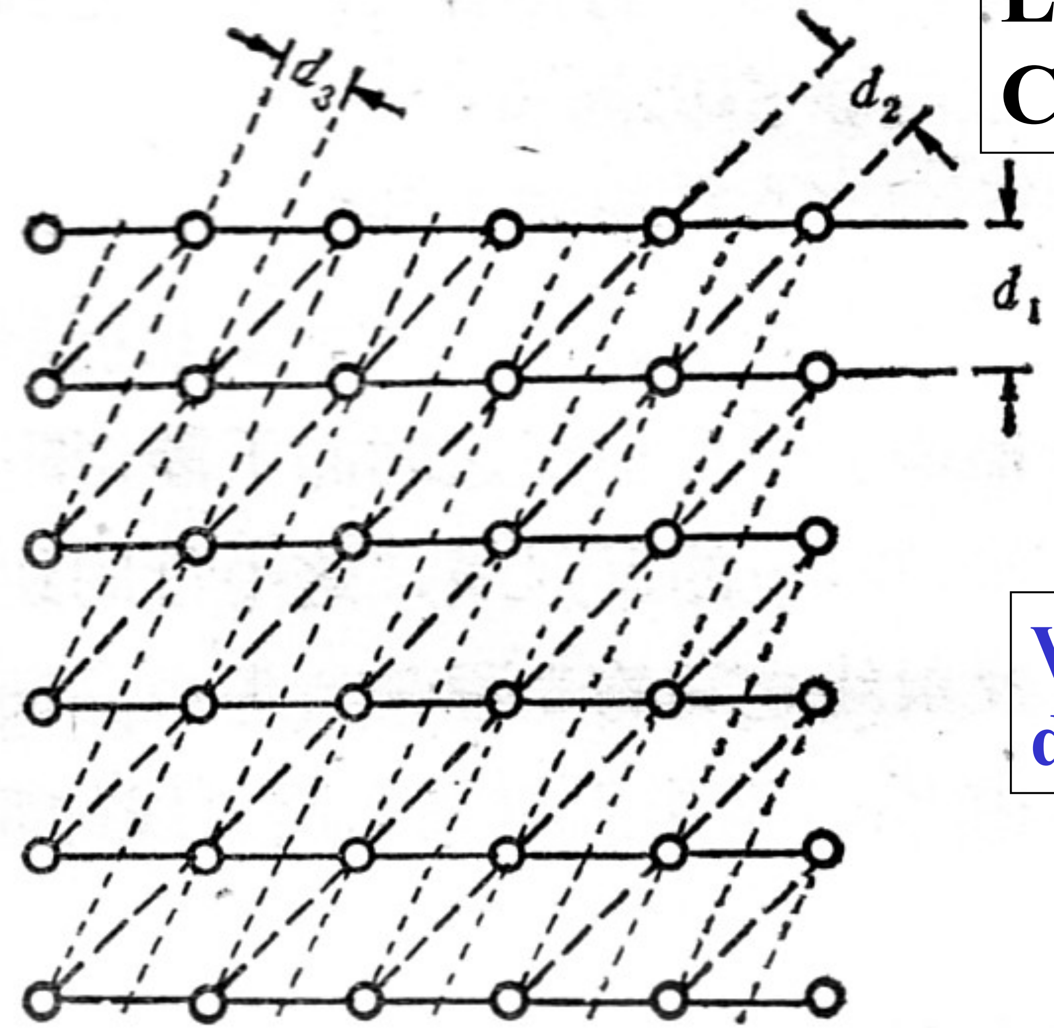
unPL waves strike on the 1st scatterer in z perpend. to xy-plan , producing y-PL wave in x direction ; after 2nd scattering,one observes y-PL waves in the z -, but not in the y- direction.

6.1.5. Diffraction of X-rays

- EM wave passes through a slit→diffraction

  (Slit size is the same order as Wavelength WL)
- Typical λ of x-rays :**0.1nm (hard to build slit)**
- **crystal: atoms (lattice) in ordered -structure**
  - 晶格常数***d*** **: crystal lattice distance is in the same order as** λ of x-rays
- crystal is a natural grating (set of slits)

Crystal planes



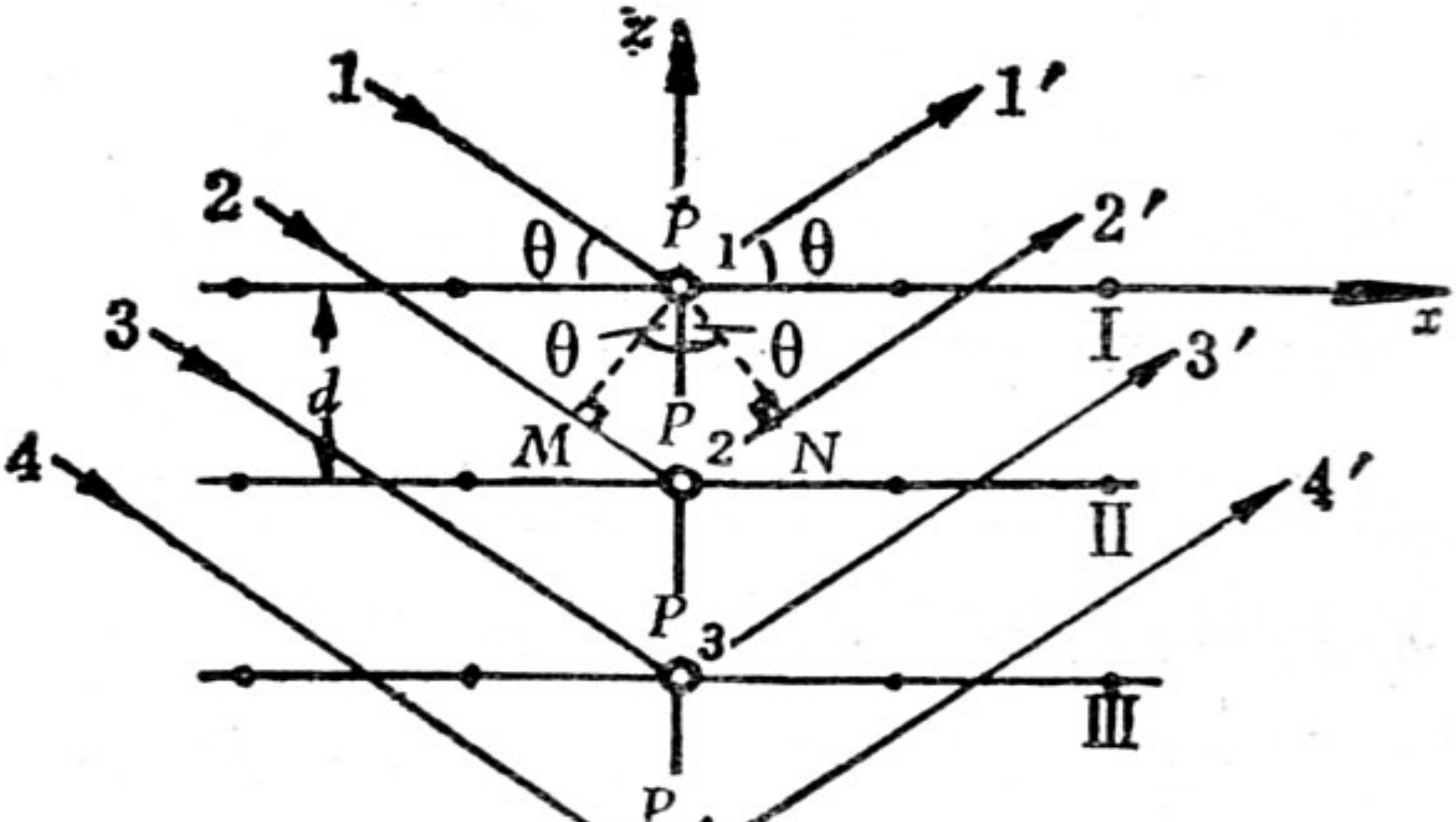
Lattices form many Crystal planes

With different lattice distances

Interference btw parallel planes

Path difference btw ray 1 & ray 2

$$\delta = \overline{MP_2} + \overline{P_2N} = d\sin\theta + d\sin\theta = 2d\sin\theta$$



Bragg Law

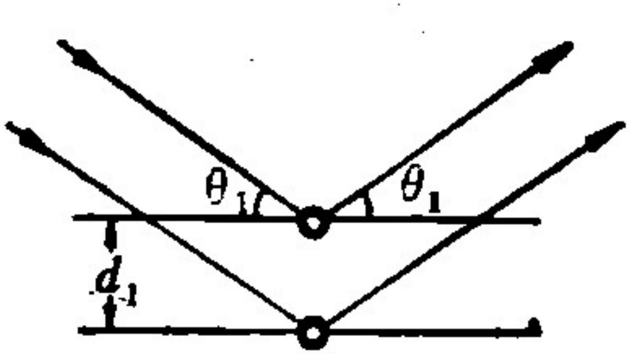
When the path difference is an integral of WL, there exists a maximum in diffraction intensity

$$\boxed{2d\sin\theta = n\lambda \qquad (n = 1, 2, 3, \ldots)}$$

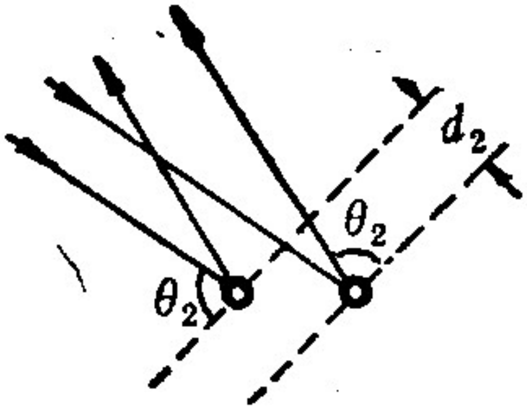
Bragg formula

- **Giving incident direction of X-ray, WL λ and cristal→a set of Bragg equations**
  - **Many sets of crystal planes**
  - **One set of crystal palne→one Bragg Equation**

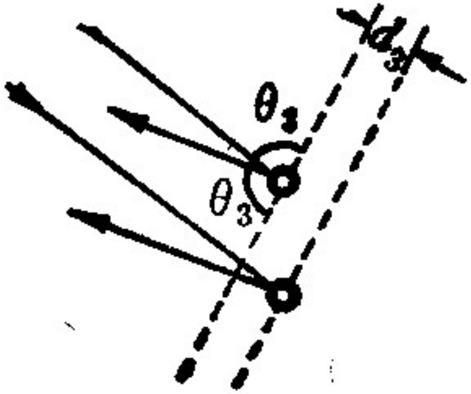
$$2d_1 \sin\theta_1 = n_1\lambda$$



$$2d_2 \sin\theta_2 = n_2\lambda$$



$$2d_3 \sin\theta_3 = n_3\lambda$$



Application of X-ray diffraction

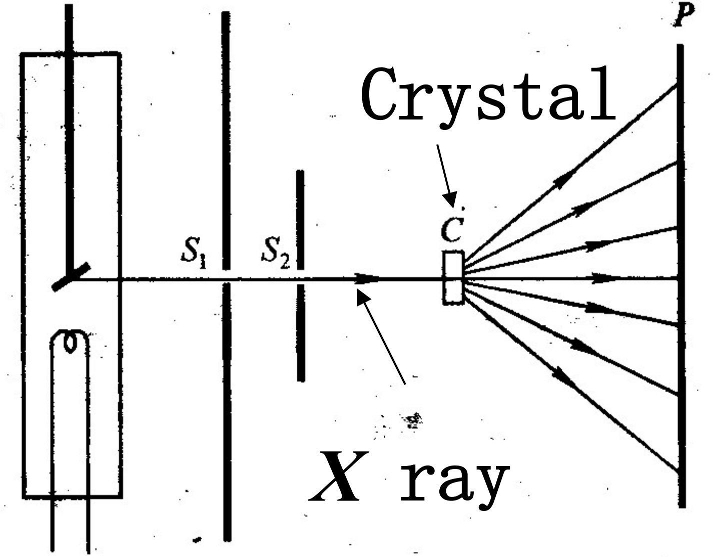
- Giving θ, λ , on can measure d —— study crystal structue and properties,

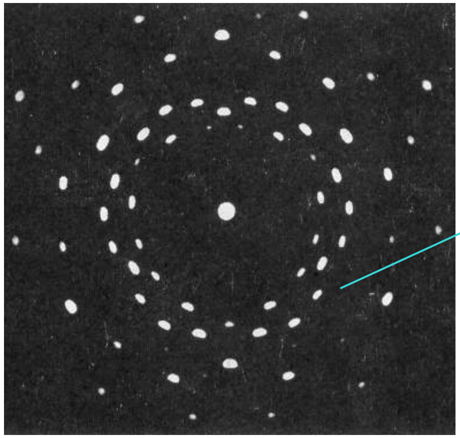
- Giving θ, d , one can measure λ —— **using** X ray light spectra, to study
- Atoms structure

$$\boxed{2d\sin\theta = n\lambda}$$

Methods to observe x-ray diffraction

- Laue film method
  - Using coutinuous WL X-ray to a single crystal
  - Giving direction, arbitary **WL**
  - **Each set of crystal planes, satisfy** $\boxed{2d\sin\theta = n\lambda}$
  - Gives a Laue spot, the position stands for the direction of the related planes →obtain all maximums。Each muximum has a bright spot

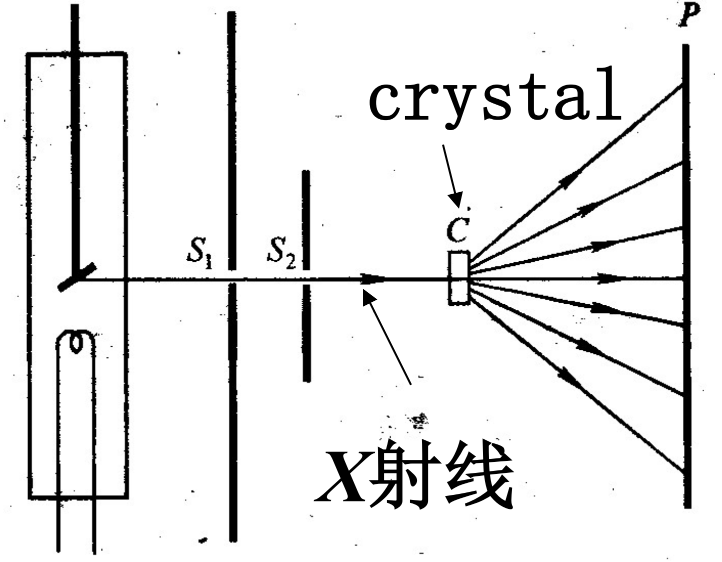


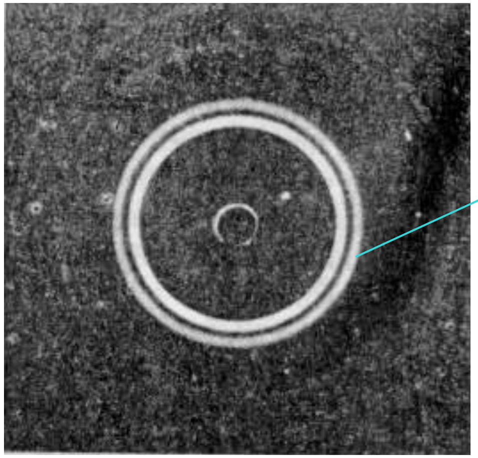


- Polycrystalline powder method
  - Using fixed WL x ray on polycrystalline powder , the planes directions are arbitrary

, the set of planes satisfing $\boxed{2d\sin\theta = n\lambda}$ gives a con-centric circle

  - →many circles←→many set of planes





Measured

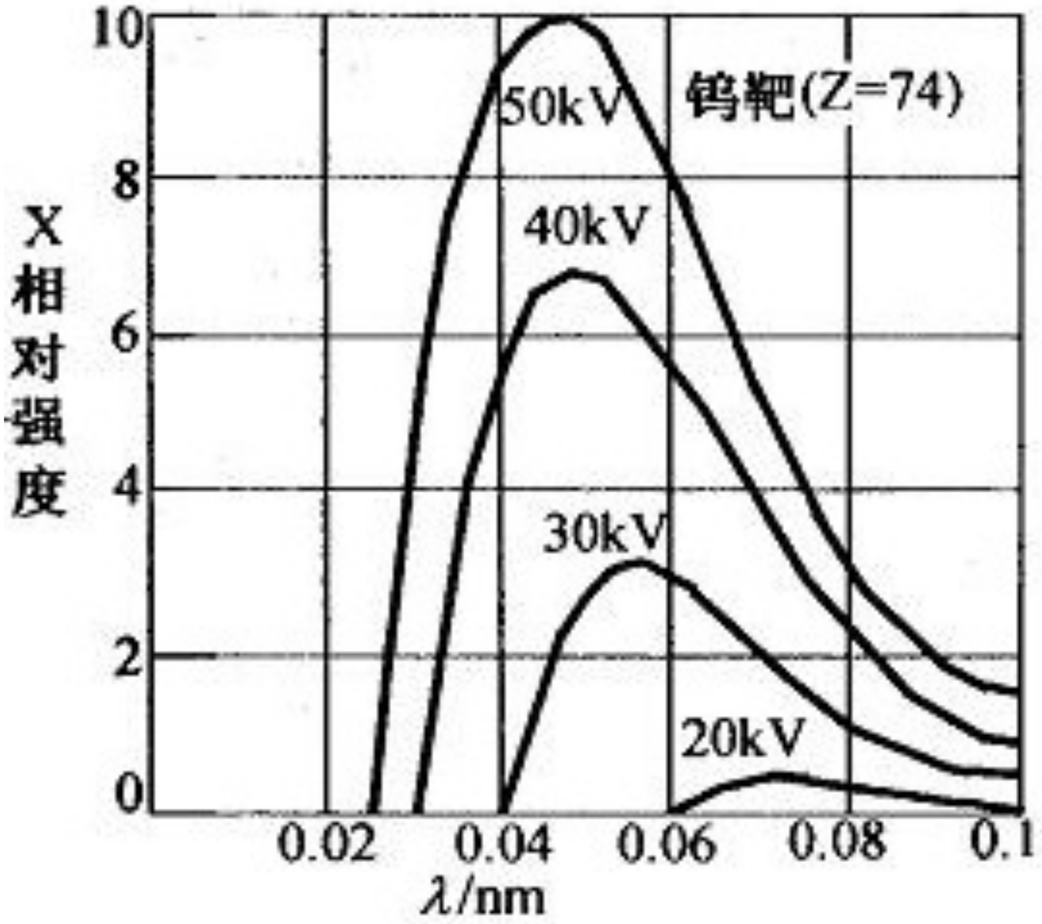
6. 2. Mechanisms for producing X-rays

6.2.1. X-ray emission spectra

X-ray emission intensity versus Wavelength

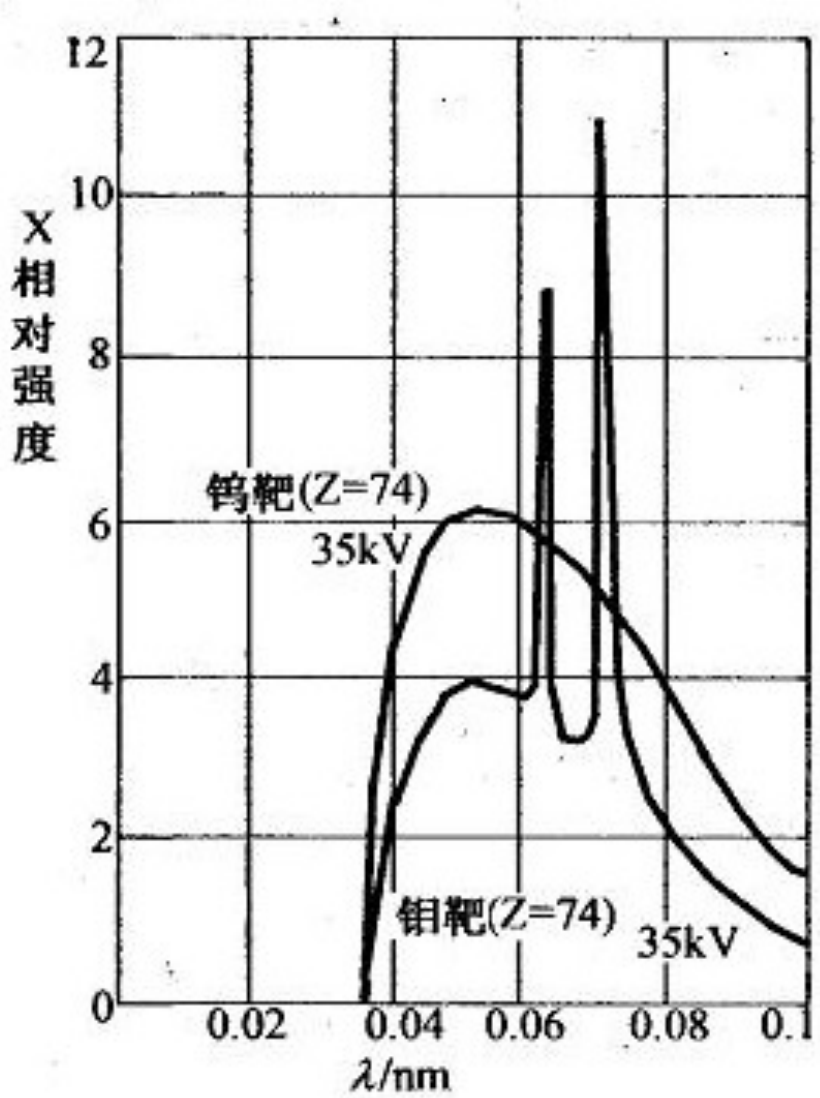
Continuous spectra:

The minuim WL depends
On applied potential



Characteristic spectra

Peaks depend on target



6.2.2. Cont. Spectra--**Bremsstrahlung**

- **Bremmsstralung**
  - Charged particles in acceleration produce EM waves
- Continuous spectra
  - X-ray tube, charged Es reach the target, their speed changes continuously under the Coulomb field of the target atoms, induce cont. x-ray spectra
- **The minimum WL depends on V**
- **X-ray emission due to de-acceleration**

- $\lambda_{\min}$ **,depends only on applied valtage** V**:**
  - **Obtained kinetic energy *eV transfers to the energy of emission photon s,*** $\lambda_{\min}$

$$eV = h\nu_{\max} = h\frac{c}{\lambda_{\min}} \to \lambda_{\min} = \frac{hc}{e}\cdot\frac{1}{V} = \frac{1.24}{V(k\mathrm{V})}\,\mathrm{nm}$$

planck *h*

Agrees with data

$\lambda_{\min}$ **confirms once more the success of quantum theory**

6.2.3. Charact spectra—transitions of the inner shell electrons

- **Threshold energy or ionization energy:**

for removing one e from n=1 shell

- **K$_\alpha$**spectra energy : energy difference from n=2 and n=1

- **Atomic light spectra is determined by the**
- **External electrons , their configuration's periodic behavior leads to its periodic**
- **K$_\alpha$ spectra determined by transitions in inner shells**

- Giving elements,charat. spectra contain several series
  - K series spectra: K_α , K_β, K_γ , … ,
  - L series spectra : L_α , L_β, L_γ , … ,
  - M series : M_α , M_β, M_γ , … ,
  - N series : N_α , N_β, N_γ , … ,

- $K_\alpha$spectra frequency Moseley formula

$$\nu_{K_\alpha} = 0.246 \times 10^{16} (Z - \sigma_K)^2 \mathrm{Hz}$$

$$\sigma_K \approx 1$$

Provides a precise measuring method of Z

Interpret Moseley formula

- when n=1 shell has a vacancy, Es in n=2 shell sense the attraction of positive charges of (Z-1) , and transit to the inner shell , producing K_α

- (n=2 → n=1) transition→ K_α spectra freq.

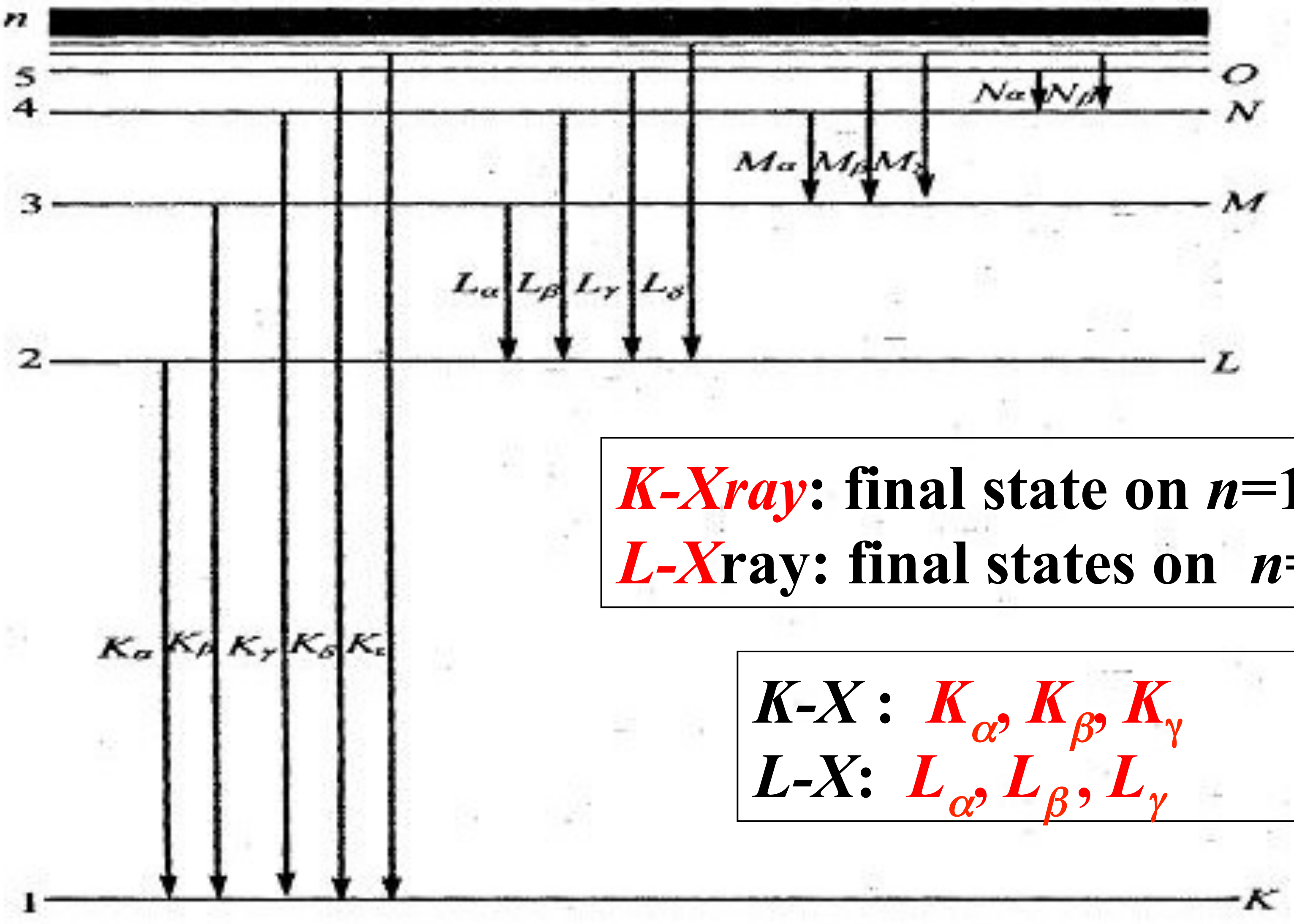
$$\nu_{K_\alpha} = \frac{c}{\lambda} = Rc\left(\frac{1}{1^2} - \frac{1}{2^2}\right)(Z-1)^2 = \frac{3}{4}Rc(Z-1)^2 = \frac{3}{4}\frac{13.6}{h}(Z-1)^2$$

$$= 0.248\times10^{16}(Z-1)^2$$

Transition e fell attraction from- **(z-1) positive charge**

Agrees with Moseley

6. 2. 4. labeling of the charact. X-ray



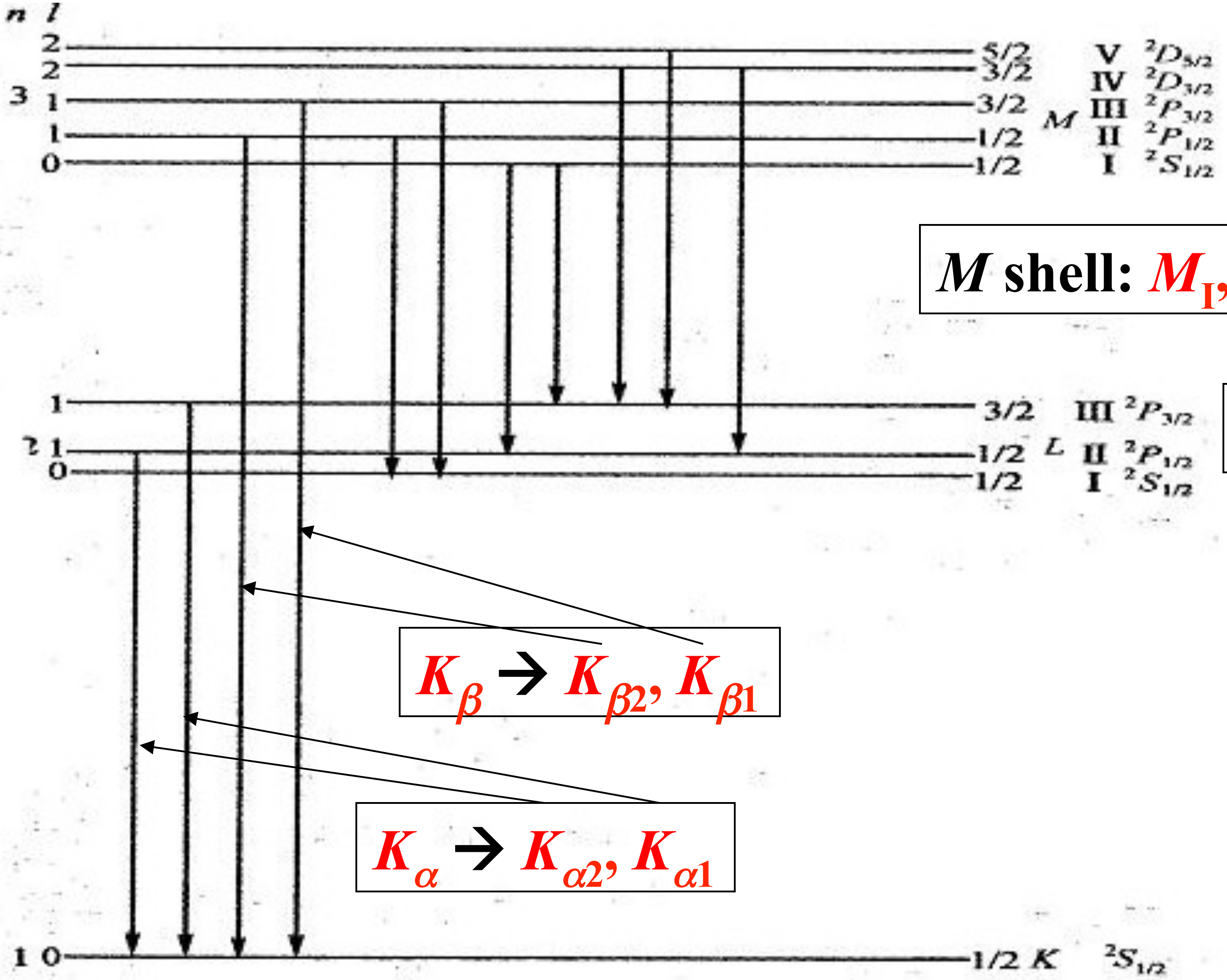
K-Xray: **final state on *n*=1(*K*) shell**

*L-X*ray: **final states on *n*=2(*L*) shell**

K-X : K_α, K_β, K_γ

L-X: L_α, L_β, L_γ

With fine structure



***M* shell: $M_{I}, M_{II}, M_{III}, M_{IV}, M_{V}$**

***L* shell: L_{I}, L_{II}, L_{III}**

$K_{\beta} \rightarrow K_{\beta 2}, K_{\beta 1}$

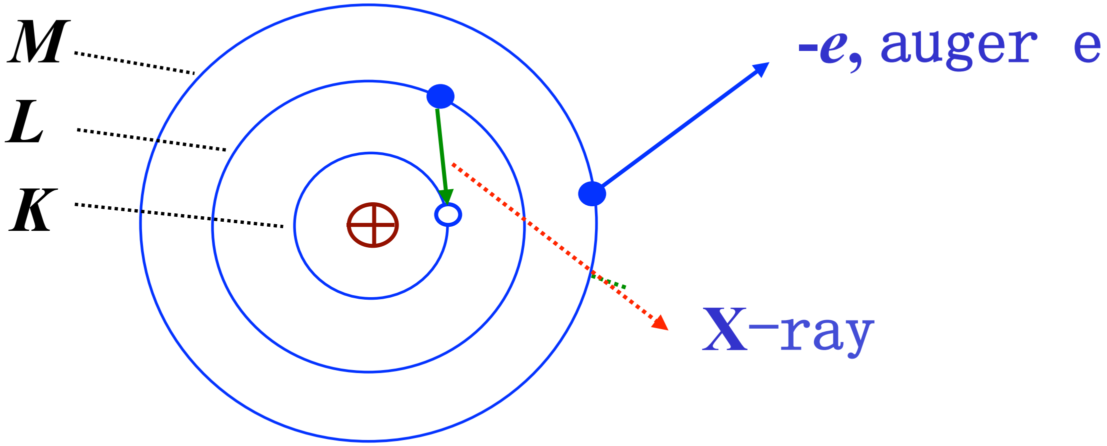
$K_{\alpha} \rightarrow K_{\alpha 2}, K_{\alpha 1}$

$\Delta L = \pm 1;$

$\Delta J = 0, \pm 1$

6.2.5. Auger Electrons

- An external shell e transits to a vacancy ,without **X-ray radiation**, tansfers its energy to another e in the same shell or outer-shell, and makes it escape from the atom



Kinetic energy :(*Φ*K –*Φ*L) -*Φ*M

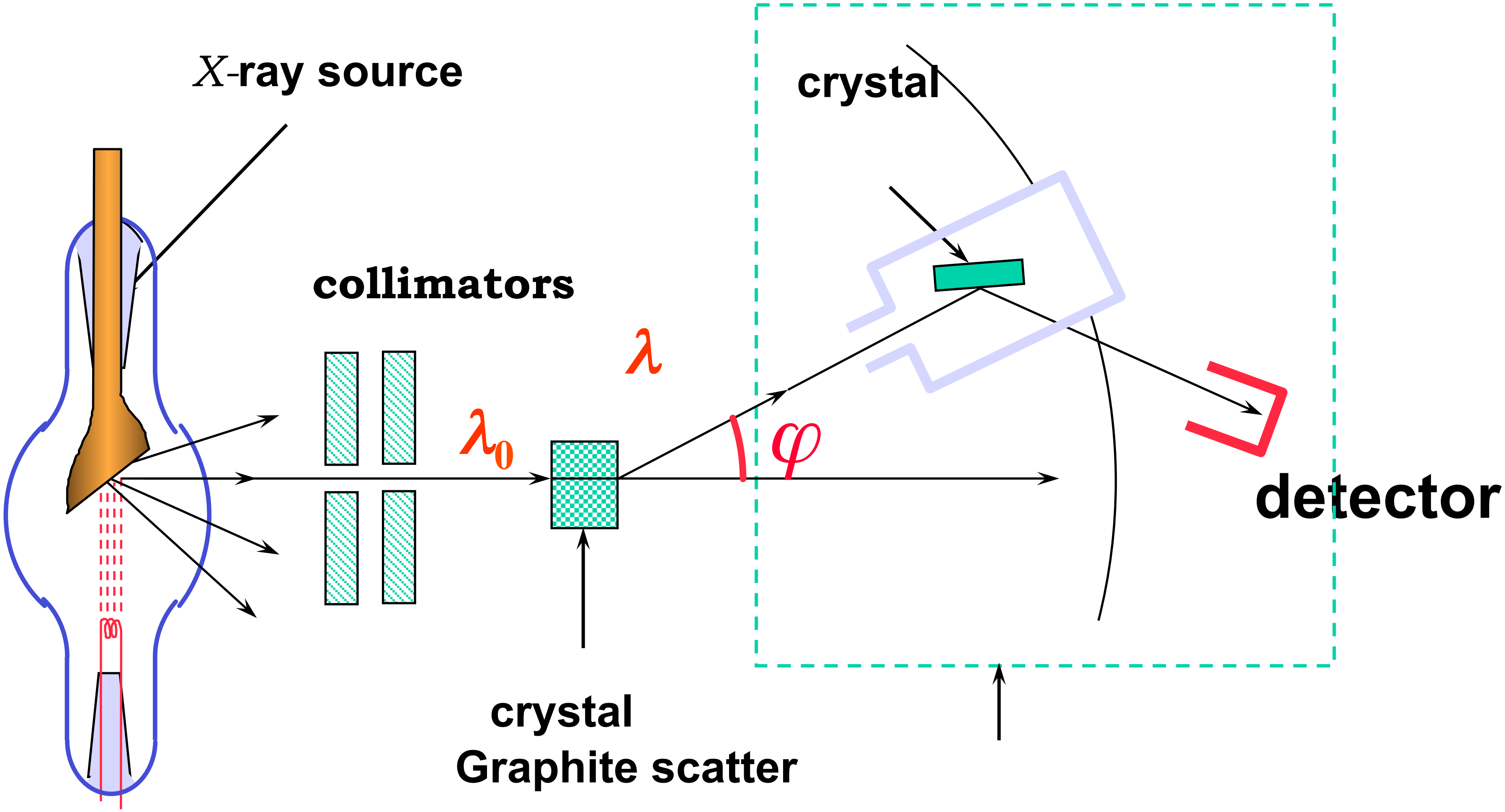
6. 2. 6. Synchrotron Radiation

- **The radiation produced by es moving in a circular paths in a sychrotron accelarator .**
  - **A new type of powerful *X-ray***
- **properties:**
  - **Width of enegy spectra 0.1～10^{4}Å** (continuous)
  - **Big power : 10 kW; X-ray tube :10 W**
  - **Highly polarized**
  - **Well-collimated in direction**

6.3. Compton Scattering

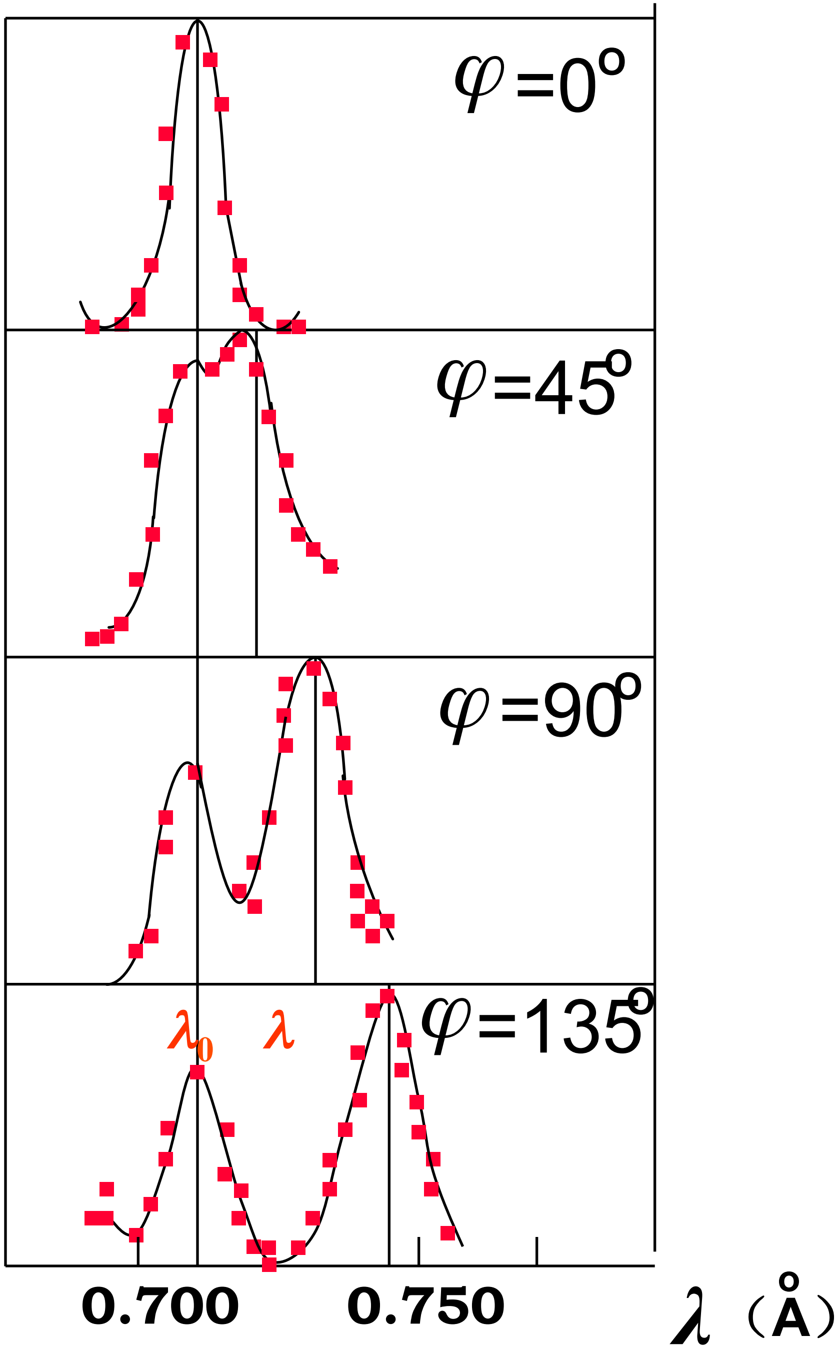
- In 1923, Compton proved the particle nature of x-ray in an experiment by scattering x-ray on matter

6. 3. 1. compton scattering exp



6. 3. 2. scatter curves

Experiment curve



Scattered x-ray has Original and longer wavelenths

Three Characters of the scattering curves

- **Besides original λ_{0}, ,there appear new bigger λ**

$$\Delta\lambda = \lambda - \lambda_0$$

- **The increase of WL, depends only on φ , not on** scatters and λ_0 $\Delta\lambda = \lambda - \lambda_0 = \lambda_c(1-\cos\varphi)$

$$\lambda_c = 0.0241\text{Å} = 2.41\times10^{-3}\text{nm}$$

- **With increase scattering angle, the intensity of original λ_0 decreased, while that of the new WL increases**

6.3.3. Classical consideration

- Classical EW theory: when a EW passes through material, the scattered EM should have the same WL as the incident one
  - incident *X* ray exerts on the Es in atoms → Es oscillate with the same freq. as the incident wave

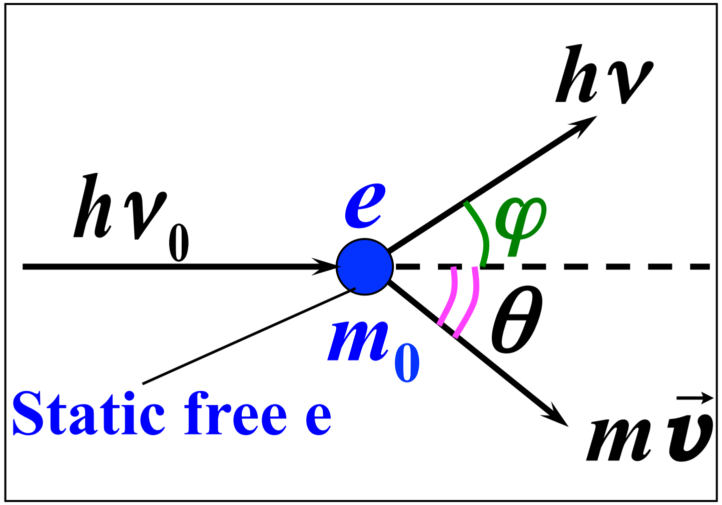
Fails to explain compton scattering

6.3.4. Quantum Explanation

- ***X rays scatter on static external Es***
  - X-ray energy $\varepsilon \sim 10^4\ \mathrm{eV}$
  - The bounding energy of external Es $\sim \mathrm{eV}$

    → can be treated as free Es
- **Elastic scattering**
- Energy and momentum conservation
- Photon tansfers part of its energy to electron, decreases its energy and hence increases its WL

- E & P conservation



$$h\nu_0 + m_0c^2 = h\nu + mc^2$$

$$\frac{h}{\lambda_0}\vec{n}_0 = \frac{h}{\lambda}\vec{n} + m\vec{v}$$

$$mc^2 = m_oc^2 + h(\nu_0 - \nu) = m_oc^2 + hc(\frac{1}{\lambda_0} - \frac{1}{\lambda})$$

$$(mc^2)^2 = (m_oc^2)^2 + 2m_oc^3h(\frac{1}{\lambda_0} - \frac{1}{\lambda}) + (hc)^2(\frac{1}{\lambda_0} - \frac{1}{\lambda})^2$$

$$(m\mathrm{v})^2 = (\frac{h}{\lambda_0})^2 + (\frac{h}{\lambda})^2 - 2\frac{h^2}{\lambda_0\lambda}\cos\varphi$$

$$(mc^2)^2 = (m_o c^2)^2 + 2m_o c^3 h\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right) + (hc)^2\left(\frac{1}{\lambda_0} - \frac{1}{\lambda}\right)^2$$

$$(m\mathrm{v})^2 = \left(\frac{h}{\lambda_0}\right)^2 + \left(\frac{h}{\lambda}\right)^2 - 2\frac{h^2}{\lambda_0\lambda}\cos\varphi$$

$$\boxed{(-)\times \mathrm{c}^2}$$

$$\cancel{(mc^2)^2\left(1-\frac{v^2}{c^2}\right)} = \cancel{(m_o c^2)^2} - 2(hc)^2\frac{1}{\lambda_0\lambda} + 2\frac{h^2c^2}{\lambda_0\lambda}\cos\varphi + 2m_o c^3 h\frac{\Delta\lambda}{\lambda_0\lambda}$$

$$\rightarrow \frac{m_o c}{h}\Delta\lambda = 1-\cos\varphi \qquad \rightarrow \boxed{\Delta\lambda = \lambda_c(1-\cos\varphi)}$$

Compton formula

$$\lambda_c = \frac{h}{m_o c} = \frac{hc}{m_o c^2} = \frac{1.24}{511} = 0.00243\mathrm{nm}$$

Agrees with data

Energy of scattered photon

$$\lambda - \lambda_0 = \underset{\substack{\uparrow \\ \boxed{*\ (hc)^{-1}}}}{\lambda_c}(1\text{-}\cos\varphi) \longrightarrow \frac{1}{h\nu} - \frac{1}{h\nu_0} = \frac{1}{m_o c^2}(1\text{-}\cos\varphi)$$

$$\rightarrow h\nu = \frac{1}{\dfrac{1}{h\nu_0} + \dfrac{1-\cos\varphi}{m_0 c^2}} = \frac{h\nu_0}{1 + \dfrac{h\nu_0}{m_0 c^2}(1-\cos\varphi)} = \frac{h\nu_0}{1+\gamma(1-\cos\varphi)}$$

$$\gamma = \frac{h\nu_0}{m_0 c^2} = \frac{hc}{m_0 c^2 \lambda_0} = \frac{\lambda_c}{\lambda_0}$$

Kinetic E of the recoiling electrons

$$h\nu_0 + m_0c^2 = h\nu + mc^2$$

$$\rightarrow E_r = mc^2 - mc_0^2 = h\nu_0 - h\nu$$

$$= h\nu_0 \frac{\gamma(1-\cos\varphi)}{1+\gamma(1-\cos\varphi)}$$

6.3.5. The physical meaning

- Compton WL of the electron

$$\lambda_c = \frac{h}{m_o c} = \frac{hc}{m_o c^2} = \frac{1.24}{511} = 0.00243\mathrm{nm}$$

- **When the E of incident photon equals the rest E of the electron**

$$h\nu_0 = m_o c^2 \to h\frac{c}{\lambda_0} = m_o c^2 \to \lambda_0 = \lambda_c = \frac{h}{m_o c}$$

- ***Wthen*** $\varphi = 90$ **,the WL difference of incident and scattered waves**

$$\Delta\lambda = \lambda_c(1\text{-}\cos\varphi) \to \varphi = 90^{\mathrm{o}}, \quad \Delta\lambda = \lambda_c \equiv \frac{h}{m_0 c}$$

Reduced Compton WL

$$\lambda'_c = \frac{\mathrm{h}}{m_0 c} = \frac{4\pi\varepsilon_0 \mathrm{h}c}{e^2}\cdot\frac{e^2}{4\pi\varepsilon_0 m_0 c^2} = \frac{r_e}{\alpha} ;\ 137 r_e$$

- **Classical radius of the electron**

$$m_0 c^2 = \frac{e^2}{4\pi\varepsilon_0 r_e} \rightarrow r_e = \frac{e^2}{4\pi\varepsilon_0 m_0 c^2} ;\ 2.8\mathrm{fm}$$

RCWL is 137 times of r_e

Some discussions

- Why $\Delta\lambda$ *is independent of scattered material* ?
  - Free electrons
- Why some of the scattered rays have its original WL?

$$m_{原子} >> m_{光子}$$

  - Coherence :$\Delta\lambda=0$
  - Non-coherence : compton scattering

- Why there is no WL shift w/ visible light scattering?

$\Delta\lambda$ **does not dependent on** λ_0, only on φ ; φ**=180,** ***the maximum of*** $\Delta\lambda = 0.0049$**nm; visible light with** $\lambda_0 \sim 500$**nm; → $\Delta\lambda/\lambda_0$ is too small**

- **Why the free electrons do not absorb , but scatter photons ?**
  - **If it do absorb photon**

$$h\nu_0 + m_0c^2 = mc^2 \quad m = m_0/\sqrt{1-v^2/c^2}$$

$$\frac{h\nu_0}{c}\hat{n}_0 = mv\hat{n}_0 \qquad \Rightarrow 1-\frac{v}{c} = \sqrt{1-\frac{v^2}{c^2}}, \quad \Rightarrow v = c$$

Significances of Comp. Scatter

- Provides strong supports for photon’s quanta hypothesis
- Proof of photon’s momentum
- Verified that Energy-momentum conservation is still valid in single event of microscopic world,

6. 3. 6. Comp. Scattering & fundamental constants

$$\Delta\lambda = \frac{h}{m_o c}(1\text{-}\cos\varphi)$$

→measure $\Delta\lambda$, φ, if one knows two of h, m_o, c

One can measure the other one

- **Measure the energy of photon**

$$E_r = h\nu_0 \frac{\gamma(1-\cos\varphi)}{1+\gamma(1-\cos\varphi)} \qquad \gamma = \frac{\lambda_c}{\lambda_0}$$

→measuring E_k, φ , λ_0 , one can obtain the energy of the incident photon

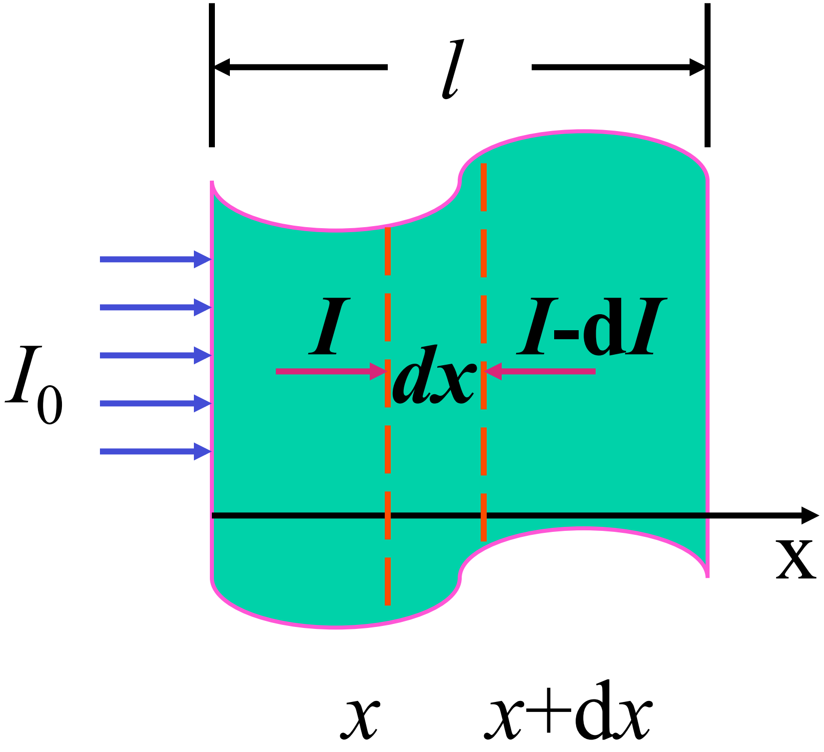
6.4. The Absorption of X-rays

- **Absorption rules**
- **Micro-mechanism of x-ray absorp.**
- **Absorption of x-ray**

6. 4. 1. Absorption rules of x-ray

- X-ray passes through material → its intensity is reduced
- $x = 0, \quad I_0$
  $x = x, \quad I(x)$
  $x = x + dx, \quad I(x) - dI$
  $- dI \propto dx\, I(x)$
  - Absorption coefficient μ

  → $- dI = \mu\, dx\, I(x)$



Integrate over $(0 \sim x)$ → Lambert-Beer rule

$$I(x) = I_0 e^{-\mu x}$$

- **Absorption length: x**

$$x\mu = 1 \rightarrow x_0 = \mu^{-1}$$

  - **After passing distance x_0, the intensity is reduced by e^{-1}**

- **Linear absorption coefficient : μ**
  - x 's unite cm, μ' s unite **cm^{-1}**

- **Absorbing matter density: ρ**

$$\mu x = \frac{\mu}{\rho} x\rho$$

Mass thickness

mg / cm^2

  - **Mass absorption coefficient $\mu\ /\rho$ cm^2/mg**
    - **$\mu\ /\rho$ does not depend on the state of absorber (gas, liquid, solid)**

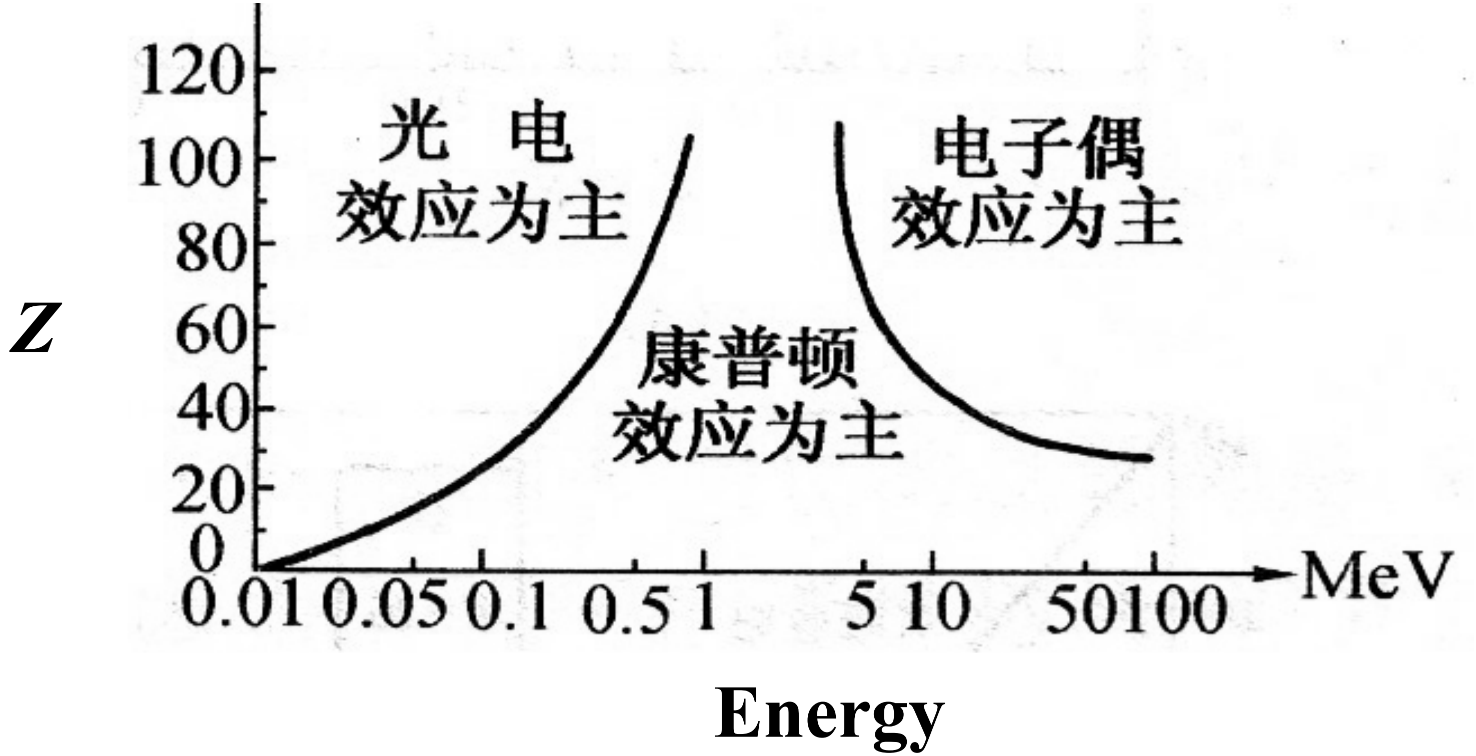
  - **Lambert-Beer rule**

$$I(x) = I_0 e^{-\frac{\mu}{\rho} x\rho}$$

6. 4. 2. Micro. Mechanism of x-ray absorption

- Photoelectric effect
  - Photons interact with bound electrons , being completely absorbed
- Compton effect
  - Photons scatter with free electrons (external shell)
- Pair production
  - When photon energy is larger than 2m_0 (1.02MeV), photon splits into e-e^{-1} pair

Relative importance of 3 major interactions btw photons and matter



$$I(x) = I_0 e^{-\frac{\mu}{\rho} x\rho}$$

$$\rightarrow \mu = \mu_{\text{光电}} + \mu_{\text{康}} + \mu_{\text{相干}}$$

6.4.3 Absorption of x-ray

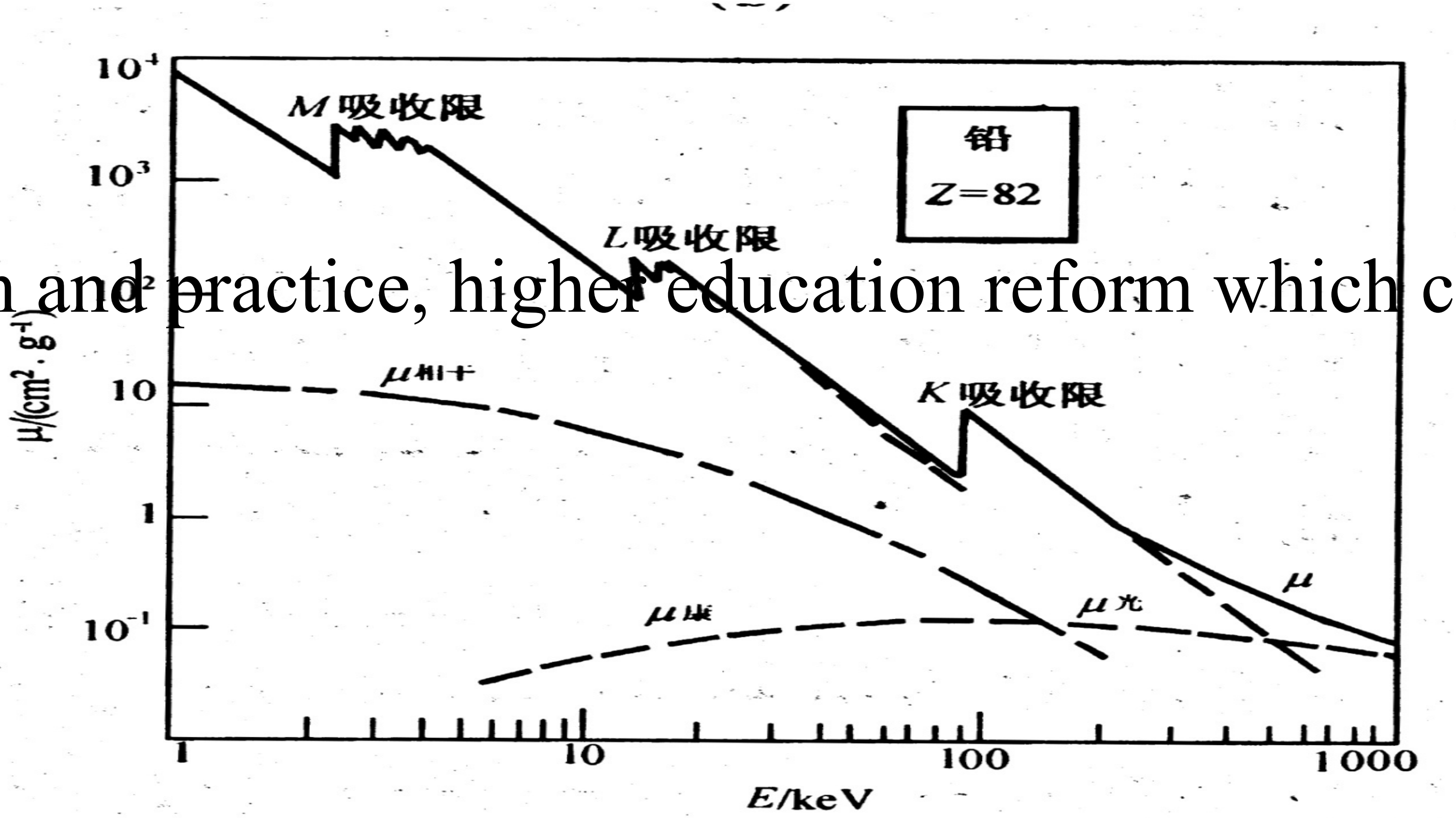
- **Matter with mixed elements**

$$\mu = \sum_j w_j \mu_j$$

- μ_j **the absorption coefficient for *j* element**
- w_j **the ratios of j element in the matter**

$$\sum w_j = 1$$

Mass absorption coefficient versus incident energy



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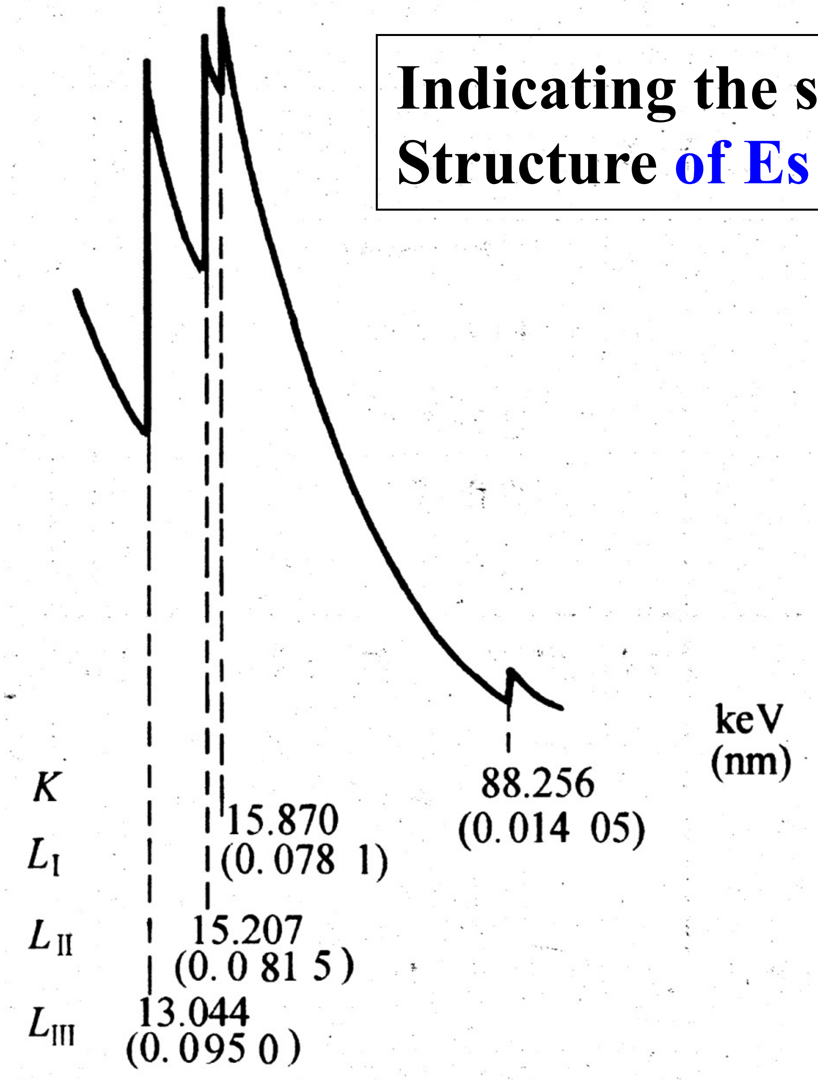
Decreasing with the increase of incident energy

Absorption edges

Peaks for sudden changing Of *μ*, corresponding to *K*、*L*、*M*…absorption lines *From energetic x photon* Removing ***K*、 *L*、*M electrons*** ***Thus* induces resonance absorption**



Indicating the shell Structure of Es